When we have lots of measurements over time, we may wish to generalize things and study trajectories. Now, instead of time steps, we are studying trends and the factors that influence them.

A citation for this work is


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The study used in this illustration examines the dynamics of post-fire recovery in California shrublands. The hypothesis being examined is that fire rejuvenates diversity of plants in the ecosystem and that following fire, there is a general decline in diversity until the next fire.

Diversity dynamics did show sort of a general decline, but with loads of plot-to-plot variation in quantity and pattern. Also, the second and fifth years showed strong upturns, raising questions as to whether there really is a trend as expected.

Figure 5. Observed values of herb species richness for the 88 plots in the dataset being examined.
Temporal data are often analyzed as either an autoregressive change or a cross-lag autoregressive model.
The SEM covariance approach to the problem of temporal dynamics often relies on using latent variables to represent latent slopes and intercepts. There is a need to set intercepts to 1.0 and random slopes are used to set a progression of time steps.

Figure 3. Simple latent trajectory model (LTM). In this model the trajectory described by observed measurements of response variable $y$ over 5 time periods can be explained by an intercept $\alpha$ and slope $\beta$. For the linear model, the values for $\lambda_j - \lambda_5 = 0, 1, 2, 3, \text{ and } 4$. 

$$\Lambda = \begin{pmatrix}
1 & 0 \\
1 & 1 \\
1 & 2 \\
\vdots & \vdots \\
1 & T-1
\end{pmatrix}.$$
There are now several major references for this model type.

General References:


lavaan implements a special function for such models called "growth". He has a tutorial on his training page.
Screenshot from Rosseel's tutorial.
Now, back to our ecological example. Here are some summary statistics.
This is a preview of the model we will develop in the subsequent pages. Note there is a good bit of machinery associated with this model type.
We start with the simplest model we can develop for the five time steps. Here the model represents the hypothesis that there is a trend over time. Note that random intercepts apply to each of the time steps (set to 1 in the command statement). A linear slope of change over time is set with the progression of 0, 1, 2, 3, and 4.
Model fit statistics show the model does converge, but has poor fit to the data.
Simple linear trajectory results structure

| Latent Variables: | Estimate | Std.Err | Z-value | P(>|z|) |
|------------------|----------|---------|---------|---------|
| i =~              |          |         |         |         |
| r1               | 1.000    |         |         |         |
| r2               | 1.000    |         |         |         |
| r3               | 1.000    |         |         |         |
| r4               | 1.000    |         |         |         |
| r5               | 1.000    |         |         |         |
| s =~              |          |         |         |         |
| r1               | 0.000    |         |         |         |
| r2               | 1.000    |         |         |         |
| r3               | 2.000    |         |         |         |
| r4               | 3.000    |         |         |         |
| r5               | 4.000    |         |         |         |

| Covariances:      | Estimate | Std.Err | Z-value | P(>|z|) |
|-------------------|----------|---------|---------|---------|
| i ~~              |          |         |         |         |
| s                 | -9.684   | 5.291   | -1.830  | 0.067   |

This slide and the next show results.
### Simple linear trajectory results structure (cont.)

#### Intercepts:

|    | Estimate | Std.Err | Z-value | P(|z|) |
|----|----------|---------|---------|--------|
| r1 | 0.000    |         |         |        |
| r2 | 0.000    |         |         |        |
| r3 | 0.000    |         |         |        |
| r4 | 0.000    |         |         |        |
| r5 | 0.000    |         |         |        |
| i  | 43.953   | 1.386   | 31.708  | 0.000  |
| s  | -3.991   | 0.321   | -12.430 | 0.000  |

#### Variances:

|    | Estimate | Std.Err | Z-value | P(|z|) |
|----|----------|---------|---------|--------|
| r1 | 135.838  | 24.450  | 5.556   | 0.000  |
| r2 | 36.455   | 9.018   | 4.042   | 0.000  |
| r3 | 63.882   | 11.439  | 5.585   | 0.000  |
| r4 | 39.726   | 8.180   | 4.857   | 0.000  |
| r5 | 54.884   | 12.420  | 4.419   | 0.000  |
| i  | 126.143  | 26.208  | 4.813   | 0.000  |
| s  | 2.443    | 1.594   | 1.533   | 0.125  |

Additional results.
And we can request modification indices in order to see some possible modifications to consider. However, in this case, we follow some initial ideas first.

Field observations suggested a carryover effects from year1 to year2 and from year2 to year 3. This is not exactly what is suggested by the mod indices, but what we will consider first.
Simple linear trajectory with autoregressive effects.

```r
### Model 102: Include autoregressive effects
mod.102 <- '
# intercept and slope with fixed coefficients
i =~ 1*r1 +1*r2 +1*r3 +1*r4 +1*r5
s =~ 0*r1 +1*r2 +2*r3 +3*r4 +4*r5
# autoregressive effects
r2 ~ r1
r3 ~ r2'

fit.102 <- growth(mod.102, data=dat2)
```

Autoregressive effects are added to the code.
Simple linear trajectory with autoregressive effects.

```
> print(fit.102)
lavaan (0.5-20) converged normally after 93 iterations

  Number of observations                 88
  Estimator                             ML
  Minimum Function Test Statistic       34.215
  Degrees of freedom                    8
  P-value (Chi-square)                  0.000
```

Model discrepancy dropped from 50.3 to 34.2, a clearly significant improvement.
Model 102: modification indices

```
> subset(modindices(fit.102), mi > 3.8)
lhs op rhs mi epc sepc.lv sepc.all sepc.nox
  3  i =~ r3 5.832 0.403 4.192 0.334 0.334
  4  i =~ r4 7.200 -0.053 -0.550 -0.050 -0.050
  5  i =~ r5 5.476 0.066 0.684 0.057 0.057
  8  s =~ r3 4.733 -2.148 -2.758 -0.220 -0.220
  9  s =~ r4 8.004 0.619 0.795 0.072 0.072
 10  s =~ r5 6.254 -0.783 -1.005 -0.084 -0.084
 22  r3 ~1 13.145 11.584 11.584 0.923 0.923
 23  r4 ~1 9.009 -2.611 -2.611 -0.236 -0.236
 24  r5 ~1 5.707 2.975 2.975 0.247 0.247
 27  r1 =~ r2 14.838 61.378 61.378 0.325 0.325
 28  r1 =~ r3 6.200 -28.577 -28.577 -0.144 -0.144
 31  r2 =~ r3 5.029 -18.318 -18.318 -0.122 -0.122
```

Suggesting an error correlation. Dicey in this case, but worth trying.

Mod indices suggest an error correlation.
Simple linear trajectory with autoregressive effect and error correlation.

```r
### Model 103: Include error correlation
mod.103 <- '
# intercept and slope with fixed coefficients
i =~ 1*r1 +1*r2 +1*r3 +1*r4 +1*r5
s =~ 0*r1 +1*r2 +2*r3 +3*r4 +4*r5
# autoregressive effects
r3 ~ r2
r2 ~ r1
# error correlation
r1 ~ r2'

fit.103 <- growth(mod.103, data=dat2)
```

Code for adding the error correlation.
Simple linear trajectory with autoregressive effect.

> print(fit.103)
> lavaan (0.5-20) converged normally after 105 iterations

<table>
<thead>
<tr>
<th>Number of observations</th>
<th>88</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimator</td>
<td>ML</td>
</tr>
<tr>
<td>Minimum Function Test Statistic</td>
<td>22.390</td>
</tr>
<tr>
<td>Degrees of freedom</td>
<td>7</td>
</tr>
<tr>
<td>P-value (Chi-square)</td>
<td>0.002</td>
</tr>
</tbody>
</table>

> fitMeasures(fit.103, "gfi")

<table>
<thead>
<tr>
<th>gfi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.986</td>
</tr>
</tbody>
</table>

Modification indices do not suggest any reasonable additions to make. So, we accept Model 103 for now. Model fit was not too bad and GFI = 0.986

Indications are there are still some imperfections in the model. Like other latent variable models, this type is a bold prediction that seeks generality over close fit. GFI suggests that fit is pretty good.
Model 3 results

| Regressions: | Estimate | Std.Err | Z-value | P>|z|) |
|-------------|----------|---------|---------|------|
| r3 ~ r2     | 0.105    | 0.024   | 4.403   | 0.000|
| r2 ~ r1     | 0.031    | 0.024   | 1.311   | 0.190|

| Covariances:     | Estimate | Std.Err | Z-value | P>|z|) |
|------------------|----------|---------|---------|------|
| r1 ~ r2     | 57.666   | 19.034  | 3.030   | 0.002|
| i ~ s        | 12.826   | 6.081   | 2.109   | 0.035|

| Intercepts:     | Estimate | Std.Err | Z-value | P>|z|) |
|-----------------|----------|---------|---------|------|
| r1              | 0.000    |         |         |      |
| r2              | 0.000    |         |         |      |
| r3              | 0.000    |         |         |      |
| r4              | 0.000    |         |         |      |
| r5              | 0.000    |         |         |      |
| i               | 41.918   | 1.580   | 26.532  | 0.000|
| s               | -3.651   | 0.395   | -9.249  | 0.000|

Autoregressive effect from time 2 to 3 is supported, but from time 1 to 2 not supported.
This figure again shows where we are going, at least in part.
Building out the network of structural effects.

```r
### Model 104: Add time-invariant covariates to Model 103
mod.104 <- '
# intercept and slope with fixed coefficients
i =~ 1*r1 +1*r2 +1*r3 +1*r4 +1*r5
s =~ 0*r1 +1*r2 +2*r3 +3*r4 +4*r5
# autoregressive effects
r3 ~ r2
r2 ~ r1
# error correlation
r1 ~~ r2
# time-invariant effects of abiotic conditions
i ~ abio
# fire severity effects
r1 ~ fire +abio
fire ~ age +abio'
```
Model 4 fit.

```r
> fit.104 <- growth(mod.104, data=dat2)
Warning message:
In lav_partable_check(lavpartable, categorical =
categorical, warn = TRUE) :
  lavaan WARNING: missing intercepts are set to zero:
  [fire]

> print(fit.104)
lavaan (0.5-20) converged normally after 97
iterations

  Number of observations                        88
  Estimator                                     ML
  Minimum Function Test Statistic                45.603
  Degrees of freedom                            21
  P-value (Chi-square)                          0.001

> fitMeasures(fit.104, "gfi")
gfi
  0.996
```

Non-fatal warning.
Abiotic favorability effect on the intercept, as well as the other added effects are supported.
This is now the tentative model for richness. Included here, though not shown in the code, is a varying annual precipitation effect that was quite important.