

# Basic lavaan Syntax Guide<sup>1</sup>

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Last modified: 17.02.15

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<sup>1</sup> For the latest ultimate reference by lavaan creator, Ives Rosseel, go to <http://lavaan.ugent.be/tutorial/tutorial.pdf>. Also, additional functions are included in the semTools package: <http://ftp5.gwdg.de/pub/misc/cran/web/packages/semTools/semTools.pdf>.

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## 1. Getting Started [[top](#)]

### A few basic points:

Lavaan is an R package for classical structural equation modeling (SEM).

An elementary introduction to SEM designed for those in the natural sciences can be found in Grace (2006). Another treatment for biologists with slightly different emphases has been written by Shipley (2000). For first time users in the social sciences, Kline's (2010) book provides an good entry-level treatment. Technical fundamentals for classic SEM are presented in Bollen (1989). A new handbook is Hoyle (2012).

Links to documentation on lavaan can be found at the lavaan site: <http://lavaan.ugent.be/>. Included at that site is a more extensive introduction "lavaanIntroduction.pdf" and a technical manual "lavaanIntroduction.pdf".

Lavaan is generally updated fairly frequently, so it is good to keep your version of R up to date. You can download the latest version of R from: <http://cran.r-project.org/>.

The lavaan package is currently still a beta-version package and not considered complete. That said, it is approaching the functionality of some commercial packages.

One feature of lavaan is that it does not require you to be an expert in R. You do need to know how to import datasets into R and how to execute commands. You also need to know how to install and load packages. The lavaan syntax is simple and requires only general background knowledge, not a deep familiarity with the R language.

The numerical results of the lavaan package are typically very close, if not identical, to the results of the commercial package Mplus. If you wish to compare the results with those obtained by other SEM packages, there are options available for doing so.

In this presentation, as is common in the biometric tradition of structural equations, the inclusion of latent variables in models is considered an advanced topic and covered later.

This presentation focuses on the lavaan command language and does not attempt to provide theoretical background or interpretational information about SEM.

## 2. Types of lavaan Commands [\[top\]](#)

There are three types of command statements to use when working with lavaan, (a) specification statements, (b) estimation statements, and (c) statements for extracting results. The reader should be aware that there are additional steps in the SEM process, particularly leading from theory to model specification and also following the extraction of results (Grace et al. 2010, Grace et al. 2012). Here is a preview of the three main lavaan commands, which will be explained subsequently.

```
### Preview of three types of lavaan commands
### Command type 1: specify model by declaring an object
model <- 'y1 ~ x1 + x2      # model defined in quotes
          y2 ~ y1 + x2'

### Command type 2: estimate parameters for the model
model.est <- sem(model, data=data.mod1) # sem command is used

### Command type 3: extract results from estimated model object
summary(model.lav2.est, rsq=T)      # summary of results
```

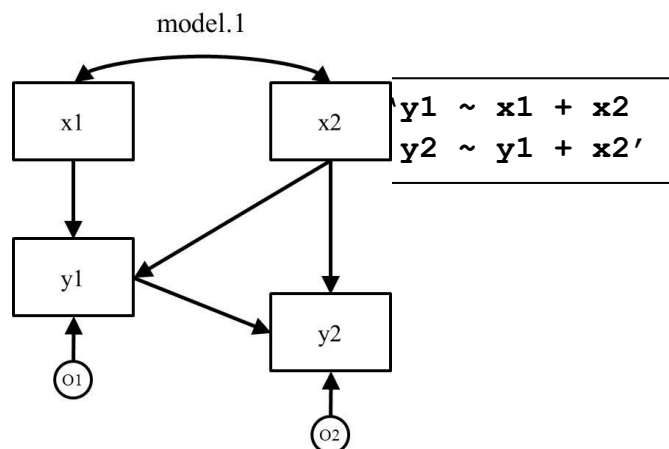
### a. Specification of a Model [\[top\]](#)

At the heart of the lavaan package is the model 'syntax'. The model syntax is a description of the model to be estimated. In this section, I briefly present the lavaan model syntax for modeling with observed variables. More syntax will be introduced in later sections.

In the R environment, a regression formula has the following form:

$$y \sim x1 + x2$$

In lavaan, a typical model is simply a set (or system) of equations contained within quotation marks. Here is a model (model.1) and its syntax:



Note that the equations (there are two in this example) are “string literals”, i.e., by placing them in quotes we make them essentially character statements. Lavaan interprets the statements in their parts, recognizing that there are three variables (y1, x1, and x2) and two operators (~, +) in the first literal and three variables (y2, y1, and x2) and two operators (~, +) in the second as well. Also note exogenous variables are allowed to correlate by default in lavaan.

The four basic types of specification operators in lavaan are:

formula type	operator	operator stands for
regression	~	“regressed on”
correlation	~~	“correlated with”
intercept	~ 1	“estimates intercept”
latent variable definition	=~	“is measured by”
create a composite	<~	“is caused by”

## b. Estimation/Fitting [\[top\]](#)

Lavaan has command statements for estimating different types of models. The most basic command is “sem”. Here I show how we can estimate the parameters in the above model.

```
model.1.ests <- sem(model.1, data = data.mod1)
```

Here, “model.1” refers to the model specification assigned to the object and “data.mod1” is the name for the data object in R.

## c. Extracting Results [\[top\]](#)

There are a variety of ways of extracting results from the estimated object. Here is the most basic extraction statement.

```
summary(model.1.ests)
```

Asking for a summary of the results gives the text below. Here we can see that lavaan converged to a stable solution. Other basic information is given.

```
> summary(model.1.ests)
lavaan (0.4-12) converged normally after 40 iterations

Number of observations                90

Estimator                            ML
Minimum Function Chi-square          23.222
Degrees of freedom                    1
P-value                              0.000
```

Parameter estimates:

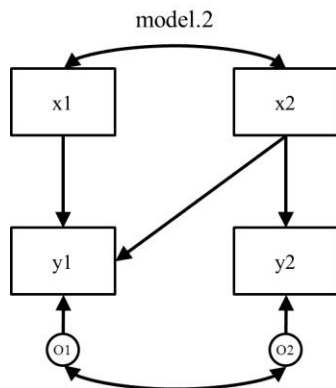
Information	Estimate	Std.err	Z-value	P(> z )	Expected Standard
Regressions:					
y1 ~					
x1	0.001	0.004	0.327	0.744	
x2	-0.083	0.019	-4.440	0.000	
y2 ~					
y1	9.910	5.083	1.950	0.051	
x2	-2.531	0.976	-2.593	0.010	
Variances:					
y1	0.080	0.012			
y2	187.212	27.908			

### 3. More Specification Options [\[top\]](#)

There are a number of additional options available that permit further specifications. Here I present several of the basic ones. More advanced commands are presented in a later section.

#### a. Correlating Errors [\[top\]](#)

Let's imagine a case where we have two endogenous responses that have a residual correlation/covariance. In the case where there is residual covariation (literally, a correlation/covariance between the prediction residuals caused by some other unspecified factor influencing both variables), we represent it as an error correlation/covariance.



```
model.2 <- 'y1 ~ x1 + x2
            y2 ~ x2
            y1 ~~ y2' #error covariance
```

It is important to note here the common convention that when a correlation is specified between two endogenous variables, it is understood that the correlation is a residual correlation and therefore, a correlation between their prediction errors (strictly speaking in causal modeling, what statisticians would call prediction errors represent other factors affecting a variable). Results presented below include a covariance between y1 and y2, which has now been requested.

---

```
> summary(model.2.ests)
lavaan (0.4-12) converged normally after 35 iterations

Number of observations                90

Estimator                            ML
Minimum Function Chi-square          22.879
Degrees of freedom                    1
P-value                               0.000

Parameter estimates:

Information                            Expected
Standard Errors                        Standard

      Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
    x1          -0.003   0.004   -0.763   0.446
    x2          -0.087   0.019   -4.643   0.000
  y2 ~
    x2          -3.363   0.896   -3.752   0.000

Covariances:
  y1 ~~
    y2           0.945   0.432    2.189   0.029

Variances:
  y1           0.081   0.012
  y2          195.119  29.087
```

---

## b. Naming Parameters [\[top\]](#)

There are a number of operations that require us to name parameters. By naming parameters, we can then specify their values or constrain their values using constraint equations.

Actually, lavaan names parameters automatically using the convention shown in output above. For example, the parameter for the effect of x1 on y1 is named “**y1 ~ x1**”.

It can be useful to name parameters in the more conventional way. Since we are used to expressing equations like this,

$$y1 = b1*x1,$$

we might prefer “b1” over “y1 ~ x1” as a parameter name. The simplest way to do this is to premultiply a predictor with the name being assigned to the parameter. Here we give the parameters in model.2 the names b1-b5. Note, parameter labels must start with a letter!

```
model.2a <- 'y1 ~ b1*x1 + b2*x2
            y2 ~ b3*x2
            y1 ~~ b4*y2'
```

Now, we get the following output, which shows both labels, original and new.

---

```
> summary(model.2a.est)
lavaan (0.4-12) converged normally after 35 iterations

Number of observations                    90

Estimator                                ML
Minimum Function Chi-square              22.879
Degrees of freedom                       1
P-value                                  0.000

Parameter estimates:

Information                               Expected
Standard Errors                          Standard

              Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
    x1      (b1)    -0.003    0.004   -0.763    0.446
    x2      (b2)    -0.087    0.019   -4.643    0.000
  y2 ~
    x2      (b3)    -3.363    0.896   -3.752    0.000

Covariances:
  y1 ~~
    y2      (b4)     0.945    0.432    2.189    0.029

Variances:
  y1              0.081    0.012
  y2            195.119   29.087
```

---

### c. Fixing Parameter Values to Specific Quantities [\[top\]](#)

There are times when we want to be able to specify that particular parameters have fixed quantitative values. Lavaan allows us to do this using various options. Here is one approach:

```
model.2b <- 'y1 ~ 0*x1 + x2
            y2 ~ x2
            y1 ~~ y2'
```

In this model statement, x1 is pre-multiplied by zero to set its value to zero. We can also accomplish this using a more elaborate and more flexible approach:

```
model.2c <- 'y1 ~ b1*x1 + x2
            y2 ~ x2
            y1 ~~ y2
            b1 == 0'
```

Now we have labeled the parameter “b1” and then assigned it a value of 0 in a separate statement. This second specification will actually result in an explicit test of the constraint.

---

```
> summary(model.2c.ests)
lavaan (0.4-12) converged normally after 158 iterations

Estimator           ML
Minimum Function Chi-square 23.329
Degrees of freedom      2
P-value                0.000

Parameter estimates:

Information                    Expected
Standard Errors                Standard

              Estimate  Std.err  Z-value  P(>|z|)
Regressions:
  y1 ~
    x1      (b1)    0.000    0.000   169.705    0.000
    x2      (b2)   -0.084    0.018   -4.611    0.000
  y2 ~
    x2      (b3)   -3.362    0.896   -3.752    0.000

Covariances:
  y1 ~~
    y2      (b4)    0.798    0.426    1.872    0.061

Variances:
  y1          0.081    0.012
  y2       195.119   29.087

Constraints:                    Slack (>=0)
  b1 - 0                          0.000
```

---



## 4. More Estimation Options [\[top\]](#)

### a. Estimating Intercepts [\[top\]](#)

By default, lavaan sets the scales for the variables to zero, placing the emphasis on the other parameters (e.g., path coefficients). One advantage for this default (along with the default of not estimating exogenous covariances) is that we don't estimate as many parameters, which is helpful when sample sizes are limited. However, there certainly are times when we want the estimates for intercepts (e.g., for generating prediction equations). Obtaining these additional parameters is easy, as it only requires overriding a default in the estimation statement. Here we revisit model.1 and ask for intercepts (for endogenous variables) using the "meanstructure" statement.

```
model.1.ests <- sem(model.1, data = data.frame,  
+ meanstructure = TRUE)
```

Actually, we could accomplish the same thing by adding command statements of the form "x1 ~ 1" to specify means and intercepts as parameters.

```
model.1a <- 'y1 ~ x1 + x2  
            y2 ~ y1 + x2  
            y1 ~ 1  
            y2 ~ 2'
```

Both approaches produce the following results:

---

	Estimate	Std.err	Z-value	P(> z )
Regressions:				
y1 ~				
x1	0.001	0.004	0.327	0.744
x2	-0.083	0.019	-4.440	0.000
y2 ~				
y1	9.910	5.083	1.950	0.051
x2	-2.531	0.976	-2.593	0.010
Intercepts:				
y1	1.004	0.233	4.318	0.000
y2	53.936	6.925	7.788	0.000
Variances:				
y1	0.080	0.012		
y2	187.212	27.908		

---

## b. Obtaining Estimates of Correlations/Covariances Between Exogenous Variables [\[top\]](#)

Lavaan follows the convention that the exogenous correlations/covariances are not estimated, but instead are taken as pre-estimated in the covariance matrix. This means, if we want to know what the covariances or correlations are between exogenous variables (and we will), we need to obtain them from the data or ask that they be estimated (override the default specification). All we need to do is include an additional statement, “fixed.x=FALSE”. Here we return to model.2 and simply ask for the x (exogenous) variables to be freely estimated instead of being fixed at the values found in the covariance matrix (e.g., “fixed.x=FALSE”).

```
#estimating the model
model.2d.ests <- sem(model.2, data = data.mod1, fixed.x=FALSE)
```

Now we obtain an estimate of the covariance in our lavaan output, as shown in bold below.

---

	Estimate	Std.err	Z-value	P(> z )
Regressions:				
y1 ~				
x1	-0.003	0.004	-0.763	0.446
x2	-0.087	0.019	-4.643	0.000
y2 ~				
x2	-3.363	0.896	-3.752	0.000
Covariances:				
y1 ~~				
y2	0.945	0.432	2.189	0.029
<b>x1 ~~</b>				
<b>x2</b>	<b>-2.651</b>	<b>1.352</b>	<b>-1.961</b>	<b>0.050</b>
Variances:				
y1	0.081	0.012		
y2	195.119	29.087		
x1	58.313	8.693		
x2	2.700	0.402		

---

We could have obtained our estimate of the exogenous covariance between x1 and x2 in R simply by using the command “cov()”

```
#estimating covariance between x1 and x2 directly
print(cov(x1, x2))
```

Of course, we can also ask for the full covariance matrix using the following statement.

```
### Ask for the full covariance matrix
print(cov(data.mod1))
```

## 5. More Options for Extracting Results [\[top\]](#)

Lavaan has numerous options for obtaining additional output from the model object. Here I focus on 5 key types of information that are commonly required for reporting results and evaluating models.

### a. Extracting the Parameter Estimates [\[top\]](#)

Lavaan has several extraction functions for pulling specific information from the estimated model object. Here I demonstrate one of the most basic, the “parameterEstimates” function. Below I will demonstrate other functions at appropriate places. For model.1, we can extract just the parameter estimates using the following syntax:

```
model.1.ests <- sem(model.1, data = data.mod1)
parameterEstimates(model.1.ests)
```

---

```
> parameterEstimates(model.1.ests)
```

	lhs	op	rhs	est	se	z	pvalue	ci.lower	ci.upper
1	y1	~	x1	0.001	0.004	0.327	0.744	-0.007	0.009
2	y1	~	x2	-0.083	0.019	-4.440	0.000	-0.119	-0.046
3	y2	~	y1	9.910	5.083	1.950	0.051	-0.052	19.873
4	y2	~	x2	-2.531	0.976	-2.593	0.010	-4.444	-0.618
5	y1	~~	y1	0.080	0.012	6.708	0.000	0.057	0.104
6	y2	~~	y2	187.212	27.908	6.708	0.000	132.513	241.911
7	x1	~~	x1	58.314	0.000	NA	NA	58.314	58.314
8	x1	~~	x2	-2.652	0.000	NA	NA	-2.652	-2.652
9	x2	~~	x2	2.700	0.000	NA	NA	2.700	2.700

---

Note we get some additional information, the confidence intervals.

### b. Standardized Estimates [\[top\]](#)

We can request standardized coefficients very easily by adding a statement to the summary command. Here we return to model.1 and request standardized parameter estimates and r-squares.

```
summary(model.1.ests, standardized=TRUE, rsq=TRUE)
```

which produces the following (only partial output shown).

	Estimate	Std.err	Z-value	P(> z )	Std.lv	Std.all
Regressions:						
y1 ~						
x1	0.001	0.004	0.327	0.744	0.001	0.032
x2	-0.083	0.019	-4.440	0.000	-0.083	-0.430
y2 ~						
y1	9.910	5.083	1.950	0.051	9.910	0.208
x2	-2.531	0.976	-2.593	0.010	-2.531	-0.277
Variances:						
y1	0.080	0.012			0.080	0.808
y2	187.212	27.908			187.212	0.830
R-Square:						
y1	0.192					
y2	0.170					

Note that the column “Std.lv” only standardizes any latent variables in the model (none in model.1, so that column is same as “Estimate” column). “Std.all” results are what we want in most cases.

Lavann has alternative methods for extracting standardize results and as stated before, these alternative methods can be very helpful when working in R because they yield objects containing key information. Here is a function “standardizedSolution” to extract standardized results.

```
> standardizedSolution(model.1.ests)
```

	lhs	op	rhs	est.std	se	z	pvalue
1	y1	~	x1	0.032	NA	NA	NA
2	y1	~	x2	-0.430	NA	NA	NA
3	y2	~	y1	0.208	NA	NA	NA
4	y2	~	x2	-0.277	NA	NA	NA
5	y1	~~	y1	0.808	NA	NA	NA
6	y2	~~	y2	0.830	NA	NA	NA
7	x1	~~	x1	1.000	NA	NA	NA
8	x1	~~	x2	-0.211	NA	NA	NA
9	x2	~~	x2	1.000	NA	NA	NA

Here, “lhs”=left-hand-side, “op”=operator, “rhs”=right-hand-side, and “est.std”=standardized estimates. Note that only the raw (unstandardized) estimates have standard errors and related properties reported.

### c. Model Fit Statistics [[top](#)]

Also of critical importance is the ability to obtain a more complete reporting of model fit statistics. Again, we have two options, one within the “summary” command and another separate function. Again for model.1,

```
summary(model.1.ests, fit.measures=TRUE)
```

yields the following:

---

```
lavaan (0.4-12) converged normally after 40 iterations
```

```
Number of observations                90
Estimator                             ML
Minimum Function Chi-square           23.222
Degrees of freedom                     1
P-value                                0.000
```

```
Chi-square test baseline model:
```

```
Minimum Function Chi-square           59.220
Degrees of freedom                     5
P-value                                0.000
```

```
Full model versus baseline model:
```

```
Comparative Fit Index (CFI)           0.590
Tucker-Lewis Index (TLI)              -1.049
```

```
Loglikelihood and Information Criteria:
```

```
Loglikelihood user model (H0)          -858.441
Loglikelihood unrestricted model (H1)   -846.830
Number of free parameters                6
Akaike (AIC)                            1728.882
Bayesian (BIC)                          1743.881
Sample-size adjusted Bayesian (BIC)     1724.945
```

```
Root Mean Square Error of Approximation:
```

```
RMSEA                                 0.497
90 Percent Confidence Interval         0.335 0.681
P-value RMSEA <= 0.05                 0.000
```

```
Standardized Root Mean Square Residual:
```

```
SRMR                                   0.134
```

---

We can also use the extractor function “fitMeasures”.

```
fitMeasures(model.1.ests)
```

which produces

---

```
> fitMeasures(model.1.ests)
      chisq          df          pvalue  baseline.chisq
      23.222         1.000         0.000         59.220
  baseline.df  baseline.pvalue          cfi          tli
      5.000         0.000         0.590         -1.049
      logl unrestricted.logl          npar          aic
     -858.441        -846.830         6.000        1728.882
      bic          ntotal          bic2          rmsea
     1743.881         90.000        1724.945         0.497
  rmsea.ci.lower  rmsea.ci.upper  rmsea.pvalue          srmr
      0.335         0.681         0.000         0.134
```

---

We can be more surgical with our function and ask for specific measures:

```
fitMeasures(model.1.ests, "bic")
```

which yields

---

```
> fitMeasures(model.1.ests, "bic")
      bic
     1743.881
```

---

There are also “BIC” and “AIC” functions.

```
> AIC(model.1.ests)
[1] 1728.882
> BIC(model.1.ests)
[1] 1743.881
>
```

Not all fit measures can be accessed in this way.

## d. Modification Indices [\[top\]](#)

Diagnosing lack of fit in models is of critical importance. In classical SEM, model fit is evaluated via discrepancies between observed and model-implied covariances, which are summarized using the above fit measures. Specific discrepancies are also of vital importance. Again, there are two approaches.

```
summary(model.1.ests, modindices=TRUE)
```

which yields:

---

	lhs	op	rhs	mi	epc	sepc.lv	sepc.all	sepc.nox
1	y1	~~	y1	0.000	0.000	0.000	0.000	0.000
2	y1	~~	y2	20.468	-53.694	-53.694	-11.331	-11.331
3	y1	~~	x1	NA	NA	NA	NA	NA
4	y1	~~	x2	0.009	851.155	851.155	1642.024	851.155
5	y2	~~	y2	0.000	0.000	0.000	0.000	0.000
6	y2	~~	x1	20.419	48.615	48.615	0.424	48.615
7	y2	~~	x2	20.468	49.621	49.621	2.010	49.621
8	x1	~~	x1	0.000	0.000	0.000	0.000	0.000
9	x1	~~	x2	0.000	0.000	0.000	0.000	0.000
10	x2	~~	x2	0.000	0.000	0.000	0.000	0.000
11	y1	~	y2	20.468	-0.287	-0.287	-13.657	-13.657
12	y1	~	x1	0.000	0.000	0.000	0.000	0.000
13	y1	~	x2	0.000	0.000	0.000	0.000	0.000
14	y2	~	y1	0.000	0.000	0.000	0.000	0.000
15	y2	~	x1	20.468	0.875	0.875	0.445	0.058
16	y2	~	x2	0.000	0.000	0.000	0.000	0.000
17	x1	~	y1	0.000	0.000	0.000	0.000	0.000
18	x1	~	y2	17.594	0.224	0.224	0.440	0.440
19	x1	~	x2	0.000	0.000	0.000	0.000	0.000
20	x2	~	y1	0.000	0.000	0.000	0.000	0.000
21	x2	~	y2	2.520	0.033	0.033	0.298	0.298
22	x2	~	x1	0.000	0.000	0.000	0.000	0.000

---

Or we can get the same information using,

```
modindices(model.1.ests)
```

This second approach gives us some additional flexibility, for example we can extract only those indices that suggest directed arrows be added (i.e., operator is ~).

```
mi <- modindices(model.1.ests)  
print(mi[mi$op == "~",])
```

Now we only get the following:

---

	lhs	op	rhs	mi	epc	sepc.lv	sepc.all	sepc.nox
1	y1	~	y2	20.468	-0.287	-0.287	-13.657	-13.657
2	y1	~	x1	0.000	0.000	0.000	0.000	0.000
3	y1	~	x2	0.000	0.000	0.000	0.000	0.000
4	y2	~	y1	0.000	0.000	0.000	0.000	0.000
5	y2	~	x1	20.468	0.875	0.875	0.445	0.058
6	y2	~	x2	0.000	0.000	0.000	0.000	0.000
7	x1	~	y1	0.000	0.000	0.000	0.000	0.000
8	x1	~	y2	17.594	0.224	0.224	0.440	0.440
9	x1	~	x2	0.000	0.000	0.000	0.000	0.000
10	x2	~	y1	0.000	0.000	0.000	0.000	0.000
11	x2	~	y2	2.520	0.033	0.033	0.298	0.298
12	x2	~	x1	0.000	0.000	0.000	0.000	0.000

---

Additional commands for filtering the long list of modifications include:

```
print(mi1[mi1$mi > 3.0,])
```

or

```
subset(modindices(cfa1.fit), mi > 4.0 & op == "~")
```

### e. Residual Covariances [\[top\]](#)

In addition to looking at modification indices, it can be useful sometimes to look at residuals.

Here we are talking about residuals in the covariance matrix, not in the data values themselves (a topic I deal with elsewhere). We can use the “resid” function for this purpose, which includes the option of looking at the standardized residuals.

```
#getting residuals
resid(model.1.ests, type="standardized")
```

which yields,

---

```
> resid(model.1.ests, type="standardized")
$cov
  y1    y2    x1    x2
y1 0.000
y2 0.000 0.004
x1 0.000 3.942 0.000
x2 0.000 0.000 0.000 0.000
```

---



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