Coefficients for Log Relations

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This module considers the interpretation of path coefficients when some or all of the variables involved are logged.

This module follows the one entitled: "SEM Essentials – Interpreting Path Coefficients", which should be studied first.

A general citation for this material is

Cohen, J., Cohen, P. West, S.G., and Aiken, L.S. 2003. Applied Multiple Regression/Correlation Analysis for the Behavioural Scienes (3rd. Ed.) Routledge, New York, USA.

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How do we interpret the effects of parameters when just the predictor variable in a relationship is logged?

There are four possible situations for a two-variable directed relation:

(1) $x \rightarrow y$

(2) $\log(x) \rightarrow y$

(3) $x \rightarrow \log(y)$

(4) $\log(x) \rightarrow \log(y)$

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I have several modules that deal with ways of interpreting parameters in models. Here I present the interpretations for three more that involve log transformations, log(x), log(y), and both log(x) and log(y) at the same time.

Advanced exercise: Search for the history behind the development of natural logarithms. You will find that the natural log describes the cumulative area under a curve where y = 1/x. This situation is one where absolute scales disappear and results are in percentages or proportions. This understanding explains the interpretations given on the next few pages for slopes or relationships involving log quantities.

Case 1: $x \rightarrow y$

For: $y = b_0 + b_1 * x$

b_0 is the expected value of y when $x = 0$.

b_1 is in the units of y/x.

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Refer to the module "SEM Essentials – Interpreting Path Coefficients" for this case.

Case 2: $\log(x) \rightarrow y$

For: $y = b_0 + b_1 * \log(x)$

b_0 is the expected value of y when $x = 1$ (i.e., $x = 1$ is the minimum benchmark for the relationship).

b_1 is the expected change in the quantity of y when $\log(x)$ changes a fixed percentage.

Note: only the use of *natural* logarithms produces coefficients that can be interpreted in terms of percentages!

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Here we consider the situation where x, the predictor, is logged, but y, the response, is not.

Case 3: $x \rightarrow \log(y)$

For: $\log(y) = b_0 + b_1 * x$

b_0 is the expected value of $\log(y)$ when $x = 0$.

b_1 is the expected <u>percentage</u> change in the quantity of $\log(y)$ when x changes a fixed amount.

Note: only the use of *natural* logarithms produces coefficients that can be interpreted in terms of percentages!

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Here we consider the situation where x, the predictor, is not logged, but y, the response, is.

Case 4: log(x) → log(y)

For: $\log(y) = b_0 + b_1 * \log(x)$

b_0 is the expected value of log(y) when $x = 1$.

b_1 is the expected percentage change in the quantity of log(y) when x changes by a percentage amount.

The coefficient b_1 is referred to as the *elasticity* in econometrics. In some fields it is very popular to log all the variables in a model so that the coefficients can be easily interpreted as percentages for scenario purposes (e.g., we can ask the question, "What will a 50% increase in transit investment create in terms of increased transit usage?")

Note: only the use of *natural* logarithms produces coefficients that can be interpreted in terms of percentages!

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Here we consider the situation where x, the predictor, is logged, and y, the response, is also logged.